練習問題の解答

9 (1) $1<3<4$ より $1<\sqrt{3}<2$

$\sqrt{3}$ の整数部分は1であるから $\sqrt{3}$ の小数部分を x とすると，

$$x=\sqrt{3}-1$$

このとき，

$$\begin{aligned}&x^2+2x-2\\&=(\sqrt{3}-1)^2+2(\sqrt{3}-1)-2\\&=3-2\sqrt{3}+1+2\sqrt{3}-2-2\\&=0\end{aligned}$$ 答

(2) $4<5<9$ より $2<\sqrt{5}<3$ となるので

$$3<\sqrt{5}+1<4$$

$\sqrt{5}+1$ の整数部分を A とすると $A=3$

また，$\sqrt{5}+1$ の小数部分を B とすると

$$\begin{aligned}B&=(\sqrt{5}+1)-A\\&=\sqrt{5}+1-3\\&=\sqrt{5}-2\end{aligned}$$

このとき，

$$\begin{aligned}AB(B+4)&=3\cdot(\sqrt{5}-2)(\sqrt{5}-2+4)\\&=3\cdot(\sqrt{5}-2)(\sqrt{5}+2)\\&=3\cdot\{(\sqrt{5})^2-2^2\}\\&=3\cdot1\\&=3\end{aligned}$$ 答

10 (1) $$\begin{aligned}\sqrt{8+2\sqrt{12}}&=\sqrt{6+2+2\sqrt{6\cdot2}}\\&=\sqrt{(\sqrt{6}+\sqrt{2})^2}\\&=\sqrt{6}+\sqrt{2}\end{aligned}$$ 答

(2) $$\begin{aligned}\sqrt{5-\sqrt{21}}&=\sqrt{\frac{10-2\sqrt{21}}{2}}\\&=\sqrt{\frac{(\sqrt{7}-\sqrt{3})^2}{2}}\\&=\frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{\sqrt{14}-\sqrt{6}}{2}\end{aligned}$$ 答

(3) $$\begin{aligned}\sqrt{11+3\sqrt{8}}&=\sqrt{11+\sqrt{72}}\\&=\sqrt{11+2\sqrt{18}}\\&=\sqrt{(\sqrt{9}+\sqrt{2})^2}\\&=\sqrt{9}+\sqrt{2}\\&=3+\sqrt{2}\end{aligned}$$ 答

11 (1) $x=\sqrt{3}+1$，$y=\sqrt{3}-1$ のとき

$$\begin{aligned}x+y&=\sqrt{3}+1+\sqrt{3}-1\\&=2\sqrt{3}\end{aligned}$$

$$\begin{aligned}xy&=(\sqrt{3}+1)(\sqrt{3}-1)\\&=(\sqrt{3})^2-1^2=2\end{aligned}$$

このとき，

$$\begin{aligned}&x^2+xy+y^2=x^2+y^2+xy\\&=(x+y)^2-2xy+xy\\&=(x+y)^2-xy\\&=(2\sqrt{3})^2-2\\&=12-2\\&=10\end{aligned}$$ 答

(2) $x=\dfrac{1}{\sqrt{3}-\sqrt{2}}$，$y=\dfrac{1}{\sqrt{3}+\sqrt{2}}$ のとき

$$\begin{aligned}x+y&=\frac{1}{\sqrt{3}-\sqrt{2}}+\frac{1}{\sqrt{3}+\sqrt{2}}\\&=\frac{\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}\\&=2\sqrt{3}\end{aligned}$$

$$\begin{aligned}xy&=\frac{1}{\sqrt{3}-\sqrt{2}}\cdot\frac{1}{\sqrt{3}+\sqrt{2}}\\&=\frac{1}{3-2}\\&=1\end{aligned}$$

よって，

$$\begin{aligned}x^2+y^2&=(x+y)^2-2xy\\&=(2\sqrt{3})^2-2\cdot1\\&=12-2\\&=10\end{aligned}$$ 答

12 (1) $$\begin{aligned}x+y&=\frac{\sqrt{2}-1}{\sqrt{2}+1}+\frac{\sqrt{2}+1}{\sqrt{2}-1}\\&=\frac{(\sqrt{2}-1)^2+(\sqrt{2}+1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}\\&=\frac{2-2\sqrt{2}+1+2+2\sqrt{2}+1}{2-1}\\&=6\end{aligned}$$ 答

$$\begin{aligned}xy&=\frac{\sqrt{2}-1}{\sqrt{2}+1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}-1}\\&=1\end{aligned}$$ 答

$$\begin{aligned}x^3+y^3&=(x+y)^3-3xy(x+y)\\&=6^3-3\cdot1\cdot6\\&=216-18\\&=198\end{aligned}$$ 答

(2) $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ より

$$\begin{aligned}3^2&=5+2\alpha\beta\\9&=5+2\alpha\beta\\2\alpha\beta&=4\\\alpha\beta&=2\end{aligned}$$

よって，

$$\begin{aligned}\alpha^3+\beta^3&=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\\&=3^3-3\cdot2\cdot3\\&=27-18\\&=9\end{aligned}$$ 答

$$\begin{aligned}\alpha^4+\beta^4&=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2\\&=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2\\&=5^2-2\cdot2^2\\&=25-8\\&=17\end{aligned}$$ 答